IMU Note for SLAM

Chanoh Park

July 6, 2018

Abstract

This document quickly introduces IMU model and building trajectory from IMU data using a MAV dataset.

1 Preliminary

1.1 Coordinate System Representation

$$ {}^W\mathbf{T}_C = \begin{bmatrix} {}^W\mathbf{R}_C & {}^W\mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix} \tag{1} $$

$$ {}^C\mathbf{T}_W = ({}^W\mathbf{T}_C)^{-1} = \begin{bmatrix} {}^C\mathbf{R}_W & -{}^C\mathbf{R}_W{}^W\mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} {}^C\mathbf{R}_W & -{}^C\mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix} \tag{2} $$

1.2 Coordinate System Transformation

$$ {}^W\tilde{\mathbf{p}} = {}^W\mathbf{T}_C{}^C\tilde{\mathbf{p}} \tag{3} $$

$$ {}^W\mathbf{p} = {}^W\mathbf{R}_C{}^C\mathbf{p} + {}^W\mathbf{t} \tag{4} $$

2 IMU Accumulation

2.1 IMU Model

$$ {}^I\mathbf{a} = {}^I\acute{\mathbf{a}} - {}^I\mathbf{R}_W{}^W\mathbf{g} - \mathbf{b}_\mathbf{a} \tag{5} $$

$$ {}^W\mathbf{g} = g\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \tag{6} $$

IMU raw acc output in body frame ${}^I\mathbf{a} \in R^3$, true acc in the body frame ${}^I\acute{\mathbf{a}} \in R^3$, body frame to world rotation ${}^I\mathbf{R}_W \in SO3$, gravity direction in the world frame ${}^W\mathbf{g} \in R^3$, IMU bias $\mathbf{b}_\mathbf{a} \in R^3$, gravity magnitude g

$$ {}^I\omega = {}^I\acute{\omega} - \mathbf{b}_\omega \tag{7} $$

IMU raw angular velocity ${}^I\omega \in R^3$, the true angular velocity ${}^I\acute{\omega} \in R^3$, bias $\mathbf{b}_\omega$

2.2 Cumulated Trajectory from IMU Reading

2.2.1 Orientation accumulation

$$ {}^{W}\mathbf{R}_{I_k} = {}^{W}\mathbf{R}_{I_0} \prod_{i=0}^{k} {}^{I_i}\mathbf{R}_{I_{i+1}} \tag{8} $$

$$ {}^{I_i}\mathbf{R}_{I_{i+1}} = e^{[{}^{I_i}\omega\delta t]_\times} \tag{9} $$

Absolute orientation of the body sampled at k_{th} time stamp w.r.t the world frame ${}^{W}\mathbf{R}_{I_k} \in SO3$, relative motion between two frames ${}^{I_i}\mathbf{R}_{I_{i+1}} \in SO3$, angular velocity at i_{th} body frame ${}^{I_i}\omega$, sampling time δt

2.2.2 Translation accumulation

$$ {}^{W}\mathbf{a}_k = {}^{W}\mathbf{R}_{I_k}{}^{I_k}\mathbf{a} \tag{10} $$

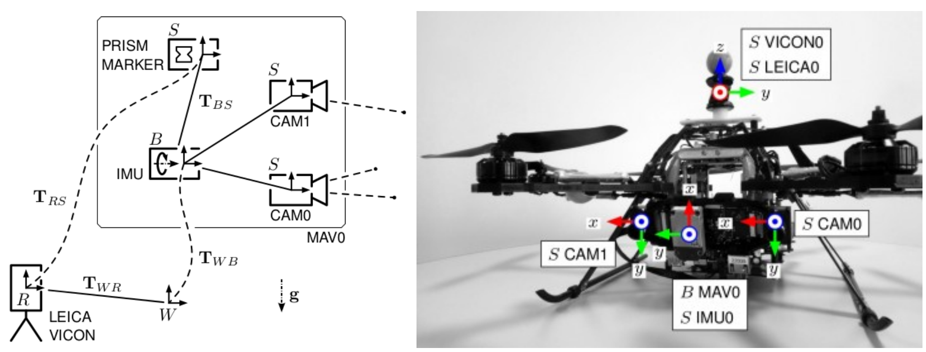
$$ {}^{W}\mathbf{v}_k = \sum_{i=0}^{k} {}^{W}\mathbf{a}_i \delta t \tag{11} $$

$$ {}^{W}\mathbf{t}_k = \sum_{i=0}^{k} {}^{W}\mathbf{v}_i \delta t \tag{12} $$

k_{th} acc, veolcity, position ${}^{W}\mathbf{a}_k, {}^{W}\mathbf{v}_k, {}^{W}\mathbf{t}_k \in R^3$

3 Real Data Example

3.1 Coordinate Systems



Defination of Gravity coordinate $[G]$, World coordinate $[W]$, IMU stationary coordinate $[I_0]$

$$ {}^{I_0}\mathbf{g} = \frac{1}{K} \sum_{i=0}^{K} {}^{I}\mathbf{a}_i \tag{13} $$

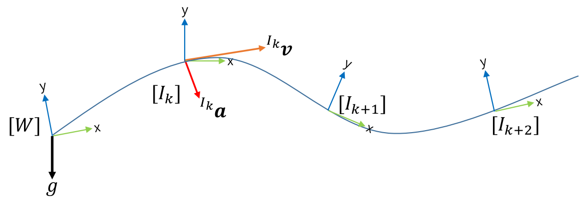


Figure 1

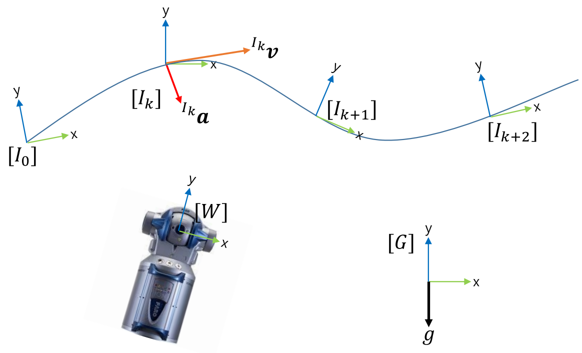


Figure 2

$$ {}^{I_0}\mathbf{R}_G = \begin{bmatrix} \mathbf{x} & \mathbf{y} & {}^{I_0}\tilde{\mathbf{g}} \end{bmatrix} \tag{14} $$

Arbitrary axis directions that are orthnormal to gravity direction $\mathbf{x}, \mathbf{y} \in R^3$, normalized gravity direction ${}^{I_0}\tilde{\mathbf{g}} \in R^3$, gravity direction during the stationary period ${}^{I_0}\mathbf{g} \in R^3$

$$ {}^{W}\mathbf{R}_{I_0} = e^{[{}^{I_0}\mathbf{r}]_\times} \tag{15} $$

$$ {}^{W}\mathbf{t}_G \tag{16} $$

$$ {}^{W}\mathbf{T}_G = \begin{bmatrix} {}^{W}\mathbf{R}_G & {}^{W}\mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix} \tag{17} $$

The world frame to the initial body frame is defined by an external pose estimation device such as vicon or laser tracker.

3.2 Initial bias estimation

$$ \mathbf{b_a} = \frac{1}{K}\sum_{i=0}^{K} {}^{I}\mathbf{a}_i - {}^{I_0}\mathbf{R}_G {}^{G}\mathbf{g} \tag{18} $$

$$\mathbf{b}_\omega = \frac{1}{K}\sum_{i=0}^{K} {}^{I_0}\omega_i \tag{19}$$

3.3 Building Trajectory by Integration

$$^G\mathbf{R}_{I_k} = {}^G\mathbf{R'}_{I_0}\prod_{i=0}^{k} {}^{I_i}\mathbf{R}_{I_{i+1}} \tag{20}$$

$$^{I_i}\mathbf{R}_{I_{i+1}} = e^{[{}^{I_i}\omega\delta t]_\times} \tag{21}$$

$$^{I_k}\mathbf{a} = {}^{I_k}\acute{\mathbf{a}} - {}^{I_k}\mathbf{R}_G {}^G\mathbf{g} - \mathbf{b_a} \tag{22}$$

$$^G\mathbf{a}_k = {}^G\mathbf{R}_{I_k} {}^{I_k}\mathbf{a} \tag{23}$$

$$^G\mathbf{v}_k = \sum_{i=0}^{k} {}^G\mathbf{a}_i\delta t \tag{24}$$

$$^G\mathbf{t}_k = {}^G\mathbf{t}'_0 + \sum_{i=0}^{k} {}^G\mathbf{v}_i\delta t \tag{25}$$

3.4 Comparing Trajectory to The Ground Truth

In MAV dataset, the ground truth is represented in the measurement device coordinate(e.g laser tracker or vicon system coordinate). Unfortunately, the z direction of this device coordinate is not coincide with the gravity direction. So, we first convert the ground truth in the world coordinate to the gravity coordinate.

$$^G\mathbf{t}'_k = {}^G\mathbf{R}_W {}^W\mathbf{t}_k \tag{26}$$

$$^G\mathbf{R'}_{I_k} = {}^G\mathbf{R}_W {}^W\mathbf{R}_{I_k} \tag{27}$$

Relative rotation between measurements

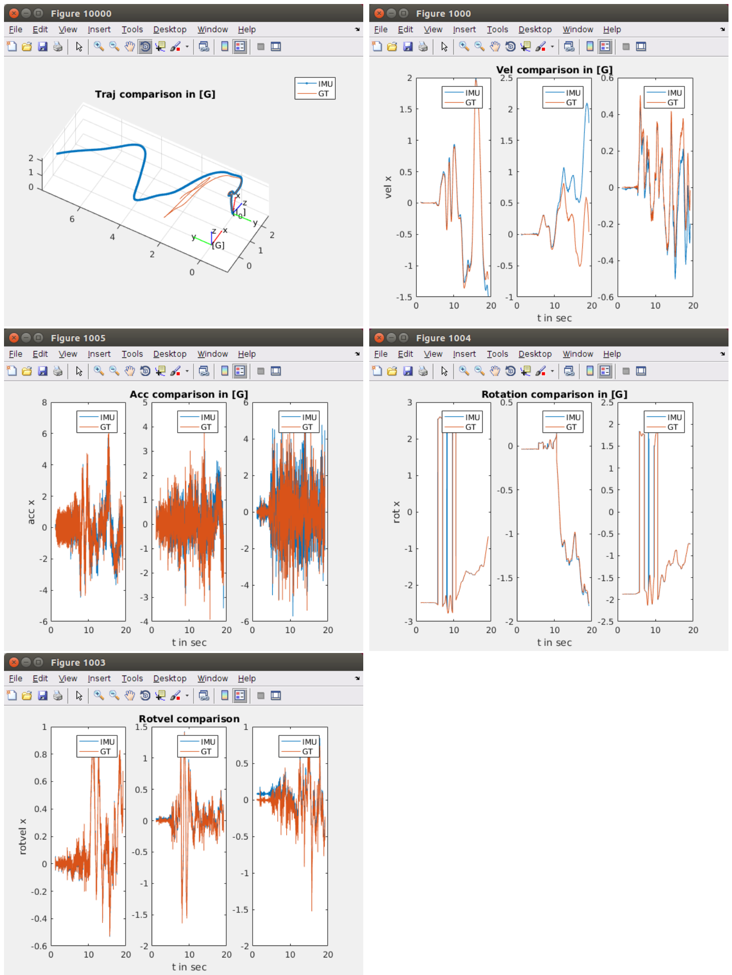
$$^{I_k}\mathbf{R}_{I_{k+1}} = {}^W\mathbf{R}_{I_k}^{-1} {}^W\mathbf{R}_{I_{k+1}} \tag{28}$$

Rotational velocity, linear velocity, linear acceration by differentiation.

$$^{I_k}\omega' = log({}^{I_k}\mathbf{R}_{I_{k+1}})/\delta t \tag{29}$$

$$^G\mathbf{v}'_k = ({}^G\mathbf{t}'_{k+1} - {}^G\mathbf{t}'_k)/\delta t \tag{30}$$

$$^G\mathbf{a}'_k = ({}^G\mathbf{v}'_{k+1} - {}^G\mathbf{v}'_k)/\delta t \tag{31}$$



Error accumulation on rotation is lower in rotation than in translation.

3.5 Appending IMU trajectory

To add new short trajectory at the end of the already existing previous trajectory by accumulating the IMU estimation , the initial rotation ${}^{G}\mathbf{R}'_{I_0}$, velocity ${}^{G}\mathbf{v}'_0$, location ${}^{G}\mathbf{t}'_0$ should be known. This is usually the case of the moving window based trajectory optimization where the coming new segment of trajectory can be only acquired by accumulating IMU.

$$ {}^{G}\mathbf{R}_{I_k} = {}^{G}\mathbf{R}'_{I_0} \prod_{i=0}^{k} {}^{I_i}\mathbf{R}_{I_{i+1}} \tag{32} $$

$$ {}^{G}\mathbf{v}_k = {}^{G}\mathbf{v}'_0 + \sum_{i=0}^{k} {}^{G}\mathbf{a}_i \delta t \tag{33} $$

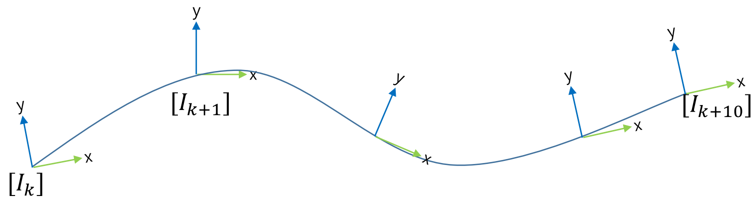


Figure 3

$$ {}^{G}\mathbf{t}_k = {}^{G}\mathbf{t}'_0 + \sum_{i=0}^{k} {}^{G}\mathbf{v}_i \delta t \tag{34}$$

3.6 IMU Pre-integration for Inertialy-constrained System

IMU pre-integration style of trajectory estimation is typically utilized in key-frame based visual-inertial odometry where all the IMU measurements are integrated to find out the relative motion ${}^{I_k}\mathbf{t}_{I_{k+10}}$, ${}^{I_k}\mathbf{R}_{I_{k+10}}$ between the two consecutive image frame. In the below example, 10 IMU measurements exist between 2 image frames. ${}^{G}\mathbf{R}_{I_k}$, ${}^{G}\mathbf{v}'_k$, ${}^{G}\mathbf{t}'_k$ should be known.

$$ {}^{I_k}\mathbf{R}_{I_{k+10}} = \prod_{i=k}^{k+10} {}^{I_i}\mathbf{R}_{I_{i+1}} \tag{35}$$

$$ {}^{I_k}\mathbf{R}_{I_{k+n}} = \prod_{i=k}^{k+n} {}^{I_i}\mathbf{R}_{I_{i+1}} \tag{36}$$

$$ {}^{G}\mathbf{R}_{I_{k+n}} = {}^{G}\mathbf{R}_{I_k} {}^{I_k}\mathbf{R}_{I_{k+n}} \tag{37}$$

$$ {}^{G}\mathbf{a}_{k+n} = {}^{G}\mathbf{R}_{I_{k+n}} {}^{I_{k+n}}\mathbf{a} \tag{38}$$

$$ {}^{G}\mathbf{v}_{k+n} = {}^{G}\mathbf{v}'_k + \sum_{i=1}^{n} {}^{G}\mathbf{a}_{k+i} \delta t \tag{39}$$

$$ {}^{G}\mathbf{t}_{k+n} = {}^{G}\mathbf{t}'_k + \sum_{i=1}^{n} {}^{G}\mathbf{v}_{k+i} \delta t \tag{40}$$

$$ {}^{I_k}\mathbf{t}_{I_{k+10}} = {}^{I_k}\mathbf{R}_G ({}^{G}\mathbf{t}_{k+10} - {}^{G}\mathbf{t}_k) \tag{41}$$

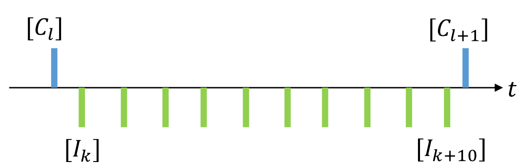


Figure 4